COURSE CODE / NAME : SQQM 2053/ DIFFERENTIAL EQUATION
DATE : 16 JANUARY 2012 (MONDAY)
TIME : 2.30 – 4.30 P.M.
VENUE : BK5 FPAU

INSTRUCTION :
1. This book script contains NINE (9) questions in TEN (10) printed pages excluding cover page.
2. Answer ALL questions in the space provided.
3. You are NOT ALLOWED to remove the exam paper from the examination hall.

MATRIC NO : ___________________________ (with word) ___________________________ (with number)
IDENTIFICATION CARD NO. :

LECTURER : ___________________________
GROUP : [ ] TABLE NO. : [ ]

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QUESTION 1 (8 MARKS)

For the following differential equation;

i. classify as Ordinary Differential Equation (ODE) or Partial Differential Equation (PDE),

ii. state the order of the differential equation,

iii. indicate whether they are linear or non-linear if it is an ODE,

iv. indicate the independent and dependent variable(s).

a) \[ x \frac{d^2y}{dx^2} + x \frac{dy}{dx} + xy = 0 \]

i. 

ii. 

iii. 

iv. 

b) \[ \frac{\partial N}{\partial E} = \frac{\partial^2 N}{\partial r^2} + \frac{1}{r} \frac{\partial N}{\partial r} + kN \]

i. 

ii. 

iii. 

iv. 

QUESTION 2 (14 MARKS)

a) For the given differential equation $y' + (2 \cos x)y = \sin 2x$,

i. show that the differential equation is linear. (1 mark)

ii. solve the differential equation. (6 marks)
b) For the following differential equation $2x + \sin y + x\cos y \frac{dy}{dx} = 0$;

i. show that the differential equation is exact.  \hspace{1cm} (2 marks)

ii. solve the differential equation. \hspace{1cm} (5 marks)
QUESTION 3 (10 MARKS)

A small town decides to conduct a fund raising drive for a futsal court whose cost is RM70,000. The initial amount in the fund is RM10,000. On the basis of past drives, it is determined that at $t$ months after the beginning of the drive, the rate $\frac{dx}{dt}$ at which money is contributed to such a fund is proportional to the difference between the desired goal of RM70,000 and total amount $X$ in the fund at that time. After one month, a total of RM40,000 is in the fund.

How much will be in the fund after three months?
QUESTION 4 (6 MARKS)
Find the general solution to the given differential equation.

a) \( y'' + 6y' + 9y = 0 \)  

b) \( \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 13y = 0 \)  

(3 marks)

QUESTION 5 (10 MARKS)
Given a second order non- homogeneous differential equation.

\[ y'' - 3y' + 2y = e^x \sin x. \]

i. Find the corresponding homogeneous solution to the differential equation  

(3 marks)
ii. Find a particular solution $y_p$. Then write the general solution to the non-homogeneous differential equation. (7 marks)
QUESTION 6 (9 MARKS)

For a second order differential equation \((x^2 - 1) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0\), it is known that \(y = x\) is a solution to it. Find the second linearly independent solution by reducing the order of the differential equation. Then write the general solution.
QUESTION 7 (7 MARKS)

Find a recurrence relation for $a_n$, where $y = \sum_{n=0}^{\infty} a_n x^n$ is a series solution to the differential equation $y + xy + y = 0$. (Hint: Give a formula for $a_{n+2}$ in terms of $a_n$ and/or $a_{n+1}$.)
QUESTION 8 (6 MARKS)

a) Use the definition to find the Laplace transform of $f(t) = e^{at}$.

(3 marks)

b) Using the linearity property of Laplace transform, determine

$$\mathcal{L} \{11 + 5e^{4t} - 6 \sin 2t\}.$$ 

(3 marks)
QUESTION 9 (10 MARKS)

Solve the following initial value problem (IVP) using Laplace transformation,

\[ y'' - 2y' - 8y = 0 \]

\[ y(0) = 3, \text{ and } y'(0) = 6. \]