FINAL EXAMINATION
FIRST SEMESTER SESSION 2011/2012

COURSE CODE / NAME : SQQM2033 ADVANCED CALCULUS
DATE : 9 JANUARY 2012 (MONDAY)
TIME : 9.00 a.m - 11.00 a.m (2 HOURS)
VENUE : BK2 (FPAU)

INSTRUCTION :

1. This book script contains TEN (10) questions in THIRTEEN (13) printed pages excluding front page.
2. Answer ALL the questions in the space provided.
3. You are NOT ALLOWED to remove the examination paper from the examination hall.

MATRIC NO : ____________________________
            (with word)
                                    (with number)

IDENTIFICATION CARD NO : ______________________

LECTURER : ____________________________

GROUP : ___    TABLE NO : ___

DO NOT OPEN THIS EXAMINATION PAPER UNTIL INSTRUCTED
QUESTION 1 (4 MARKS)

a) Give **ONE** (1) example of a geometric sequence. (1 mark)

b) Show that the following sequence is divergence. (3 marks)

\[
a_n = \left\{ \frac{5n^5 + n^3 + 2}{7n^4 + n^2 + 3} \right\}
\]
QUESTION 2 (3 MARKS)

a) State ONE (1) difference between sequence and series. (2 marks)

b) State a condition for $p$-series to be divergent. (1 mark)

QUESTION 3 (4 MARKS)

Using an appropriate test, determine whether the series $\sum_{k=1}^{\infty} \left(2 - \frac{2}{k}\right)^k$ is convergence.
QUESTION 4 (7 MARKS)

Find the interval and radius of convergence of power series \( \sum_{n=1}^{\infty} \frac{(x + 1)^n}{(-3^n)} \).
QUESTION 5 (8 MARKS)

a) Show that each of the given points lies on the polar equation, \( r = \frac{2}{1 - \cos \theta} \).

i. \( (2, \frac{\pi}{2}) \) \hspace{1cm} (2 marks)

ii. \( (-2, \frac{3\pi}{2}) \) \hspace{1cm} (2 marks)
b) Sketch the graph for \( r = 3 - 3\cos(\theta - \frac{\pi}{6}) \). (4 marks)
QUESTION 6 (7 MARKS)

a) Describe the domain and range of the function \( z(x, y) = \sqrt{y - x^2} \). (3 marks)

b) Find \( f_{xyz} \) if \( f(x, y, z) = ye^{2y} - 2xy^2z \). (4 marks)
QUESTION 7 (13 MARKS)

a) Find the linearization of \( f(x, y) = \ln(\sqrt{x^2 + 2y^2}) \) at point (1,1) and use it to approximate \( f(0.9,1.1) \). (8 marks)
b) Given $w = f(x, y)$, $x = g(r, s)$, and $y = h(r, s)$.

i. Express $\frac{\partial w}{\partial r}$ in terms of functions $x$ and $y$. (1 mark)

ii. Find $\frac{\partial w}{\partial s}$ if $w = -x^2 + y$, and $x = \cos r - \frac{r}{s}$. (4 marks)
QUESTION 8 (14 MARKS)

a) Show that the function \( g(x, y) = y^2 - x^2 + xy \) has a saddle point. (6 marks)
b) Given $f(x, y, z) = 4x^2 + y^2 + 5z^2$. Find the point on the plane $2x + 3y + 4z = 12$ at which $f(x, y, z)$ has its least value. (8 marks)
QUESTION 9 (6 MARKS)

Evaluate the iterated integral: \( \int_0^1 \int_0^\sqrt{1-v^2} \, du \, dv \).
QUESTION 10 (14 MARKS)

a) Find the area of the region $R$ by the parabola $y = x^2$ and the line $y = x + 2$.

(6 marks)
b) Evaluate \( \iiint_R z^2 e^z \, dV \) where \( R \) is the box given by

\[
0 \leq x \leq 1, 0 \leq y \leq x, -1 \leq z \leq 1
\]

(8 marks)

\[\text{END OF QUESTIONS}\]