### FINAL EXAMINATION
**FIRST SEMESTER SESSION 2011/2012**

**COURSE CODE / NAME:** SQQM2023 LINEAR ALGEBRA  
**DATE:** 7\(^{th}\) January 2012 (SATURDAY)  
**TIME:** 8.30p.m. – 10.30p.m. (2 HOURS)  
**VENUE:** BK3 (FWB) and BK4 (FWB)

**INSTRUCTION:**

1. This book script contains THIRTEEN (13) questions in TEN (10) printed pages excluding the cover page.

2. Answer ALL questions in the space provided.

3. You are NOT ALLOWED to remove the examination paper from the examination hall.

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**MATRIC NO:** ____________________________  
( with word )  
( with number )

**IDENTIFICATION CARD NO.:**  
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**LECTURER:** ____________________________

**GROUP:** [ ]  
**TABLE NO.:** [ ]

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**DO NOT OPEN THIS EXAMINATION PAPER UNTIL INSTRUCTED**

**CONFIDENTIAL**
QUESTION 1 (5 MARKS)

\[
\begin{bmatrix}
  a & 0 & 0 & 0 \\
  b & c & 0 & 0 \\
  d & e & f & 0 \\
  g & h & i & j \\
\end{bmatrix}
\]

a. Determine the value of \(a\). (2 marks)

b. Consider the following system with two variables:
\[
\begin{align*}
2x - y &= 4 \\
4x + cy &= 5
\end{align*}
\]
Determine the value of \(c\) such that Cramer’s rule cannot be used to solve the given system. Explain your answer. (3 marks)

QUESTION 2 (4 MARKS)

Consider the matrix equation \(AX = B\) given by
\[
\begin{bmatrix}
  5 & 2 \\
-15 & -6
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
b_{11} \\
b_{21}
\end{bmatrix}.
\]

i. Write the augmented matrix, \(A^* = [A|B]\). (1 mark)
ii. What is the condition on $b_1$ and $b_2$, such that the matrix equation fails to have solution? (3 marks)

QUESTION 3 (6 MARKS)

A biologist has placed three strains of bacteria (denote I, II, and III) in a test tube, where they will feed on three different food sources (A, B, and C). Each day 400 units of A, 600 units of B, and 600 units of C are placed in the test tube, and each bacterium consumes a certain number of units of each food per day, as presented in the following system.

\[
\begin{align*}
    x + 2y &= 400 \\
    2x + y + z &= 600 \\
    x + y + 2z &= 600
\end{align*}
\]

Assume that $x$, $y$, and $z$ be the numbers of bacteria of strains I, II, and III, respectively. By using Gauss elimination method, how many bacteria of three strains can coexist in the test tube and consume the food?
QUESTION 4 (5 MARKS)

Let \( p = (\sqrt{2}, 1, -1) \) and \( q = (0, 2, -2) \). Compute

i. \[ p + q \] (2 marks)

ii. \[ p \times q \] (3 marks)

QUESTION 5 (7 MARKS)

Let \( V \) be the set of all pairs of real numbers of the form \((u_1, 1)\) with the operations
\[(u_1, 1) + (v_1, 1) = (u_1 + v_1, 1)\] and
\[k(u_1, 1) = (ku_1, 1),\] where \( k \) is a scalar.

i. Is \( k(u + v) = ku + kv \) for any \( u \) and \( v \) in \( V \) and any real number \( k \)? (3 marks)
ii. Is \((k + m)\mathbf{u} = k\mathbf{u} + m\mathbf{u}\) for any \(\mathbf{u}\) in \(V\) and any real number \(k\) and \(m\)?

(3 marks)

iii. Is \(V\) a vector space? Explain your answer.

(1 mark)

**QUESTION 6 (3 MARKS)**

Is the set of all \(3\times3\) upper triangular matrices, 
\[
\begin{bmatrix}
a & b & c \\
0 & d & e \\
0 & 0 & f \\
\end{bmatrix}
\]
a subspace of the vector space, \(M_{33}\)?

Justify your answer.
QUESTION 7 (8 MARKS)

Given \( \mathbf{u} = \begin{bmatrix} 2 \\ 6 \\ 6 \end{bmatrix}, \mathbf{u}_1 = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \) and \( \mathbf{u}_2 = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix} \).

i. Write \( \mathbf{u} \) as a linear combination of \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \). (1 mark)

ii. Is \( \mathbf{u} \in \text{span} \{ \mathbf{u}_1, \mathbf{u}_2 \} \)? Justify your answer. (7 marks)
QUESTION 8 (9 MARKS)

\[
A = \begin{bmatrix}
1 & 1 & -3 & 2 \\
2 & -2 & 2 & 0 \\
-1 & 1 & -1 & 0
\end{bmatrix}
\]

Let \( A \) be the coefficient matrix of the homogeneous system \( Ax = 0 \).

i. Write the corresponding homogeneous system \( Ax = 0 \).  
   (2 marks)

ii. Solve the homogeneous system \( Ax = 0 \).  
    (5 marks)

iii. Find a basis and the dimension for the null space of \( A \).  
    (2 marks)
QUESTION 9 (4 MARKS)

Suppose that $u$, $v$ and $w$ are vectors such that $\langle u, v \rangle = 3$, $\langle u, w \rangle = 2$, $\langle v, w \rangle = -1$ and $\|v\| = 7$. Find $\langle u - 2v, v + w \rangle$.

QUESTION 10 (11 MARKS)

$\begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Let $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Find

i. all the eigenvalues of $A$.

(4 marks)
ii. all the eigenvectors of \( A \).

iii. a matrix \( P \) that diagonalizes \( A \).
QUESTION 11 (5 MARKS)

Is \( A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \) diagonalizable? Justify your answer.

QUESTION 12 (7 MARKS)

Determine whether the function \( T : R^3 \rightarrow R^2 \) given by the formula
\( T(x_1, x_2, x_3) = (x_1 + x_2 - x_3, x_1 - 2x_3) \) is a linear transformation or not. Justify your answer.
QUESTION 13 (6 MARKS)

Given \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^4 \) where \( T(x, y, z) = (x - y + 2z, x + y - z, 2x + z, 2y - 3z) \).
Find \( \ker(T) \).