FINAL EXAMINATION
FIRST SEMESTER SESSION 2011/2012

COURSE CODE / NAME : SQQM1063 (DISCRETE MATHEMATICS)
DATE : 17th JANUARY 2012 (TUESDAY)
TIME : 9:00 am – 11:30 am
VENUE : DSB K.T/WD

INSTRUCTION :

1. This exam paper contains ELEVEN (11) questions in ELEVEN (11) printed pages, excluding the cover page.
2. Answer ALL QUESTIONS on the exam paper.
3. You are NOT ALLOWED to remove the exam paper from the examination hall.

MATRIC NO : ____________________________________________ (with word) ____________________________________________ (with number)

IDENTIFICATION CARD NO. :

LECTURER : ____________________________________________

GROUP : ___________ TABLE NO. : _______________________

DO NOT OPEN THIS EXAMINATION PAPER UNTIL INSTRUCTED

CONFIDENTIAL
QUESTION 1 (8 MARKS)

Determine whether the following are true or false.

a) If $1 + 1 = 2$, then $2 + 2 = 5$
   ..............................................

b) The conjunction of two tautologies is a tautology.
   ..............................................

c) The negation of an existential statement is a universal statement.
   ..............................................

d) Counterexample is used to show that a proposition is true
   ..............................................

e) For any graph, removing any edge from a cycle, the result is a graph that is still connected.
   ..............................................

f) The chromatic number is the positive integer $k$ for which the graph is $k$-colorable.
   ..............................................

g) Every tree is a bipartite graph.
   ..............................................

h) If $T$ is a tree with 50 vertices, the largest degree that any vertex in $T$ can have is 49.
   ..............................................
QUESTION 2 (9 MARKS)

(a) Show that the proposition \((p \land q) \rightarrow (\neg p \lor q)\) is a tautology:

i. using truth table. \hspace{2cm} (4 marks)

ii. using laws of logical equivalent. \hspace{2cm} (5 marks)
QUESTION 3 (3 MARKS)

Let $L(x, y)$ be the statement “$x$ loves $y$”, where the domain for both $x$ and $y$ consists of all people in the world. Use quantifier to express each of these statements

(a) Everybody loves somebody. (1 mark)

(b) Nobody loves everybody. (1 mark)

(c) Everyone loves Harry Potter. (1 mark)

QUESTION 4 (4 MARKS)

Suppose the predicate $Q(x, y)$ means “$x + 2y = 5$” and the domain of $x$ and $y$ is all integers. Determine the truth values and counterexample of the following statements.

i. $\forall yQ(1, y)$. (2 marks)

ii. $\forall y\exists xQ(x, y)$. (2 marks)
QUESTION 5 (10 MARKS)

(a) Determine whether these arguments are valid:

Adam works part time or full time.
If Adam does not play on the team, then he does not work part time.
If Adam plays on the team, he is busy.
Adam does not work full time.
Therefore, Adam is busy.  

(4 marks)
(b) What logical error occurs in the following arguments?

If $n$ is a real number with $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 \leq 4$.

(3 marks)

(c) Using the definitions of even integer and odd integer, give a direct proof that this statement is true for all integers $n$.

If $n$ is odd, then $5n + 3$ is even.

(3 marks)
QUESTION 6 (7 MARKS)

a) Based on Table 1,

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>z</th>
<th>( F(x, y, z) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

i. Write the sum-of-products expansions that represent the function \( F(x, y, z) \). (2 marks)

ii. Show that the function \( F(x, y, z) \) can be simplified to \( (x + y)\bar{z} \). (3 marks)

b) Find the sum-of-products expansions for the Boolean function \( G(x, y) = 1 \). (2 marks)
For the purpose of energy electric saving, there is an automatic air conditioner system in lecture hall. The air conditioner should be turned on if and only if the room temperature is more than 25°C and the time is between 8 a.m. to 5 p.m. The air conditioner also will be turned on if the lecturer enters the lecture hall as all the conditions above had been ignored.

i. Write all the variables involve in the problem statement above. (3 marks)

ii. By using the variables in (i), construct a Boolean expression that represent the automatic air conditioner system. (2 marks)

iii. Draw the logic gates for the Boolean expression. (2 marks)

iv. Determine the state of the air conditioner if the room temperature is less than 25°C, the time is 10 a.m., but the lecturer does not enter the lecture hall? Justify your answer. (3 marks)
Given the graph

Determine the followings:

i. Number of vertices and edges. (2 marks)

ii. Find the degree for each vertex. (2 marks)

iii. Compare the sum of the degrees with the number of edges. How are these two numbers related. (2 marks)

iv. List all cycles in the given graph. (3 marks)
QUESTION 9 (6 MARKS)

Determine the chromatic number for the followings:

a) Bipartite graph

(2 marks)

b) Cycle

(4 marks)

QUESTION 10 (8 MARKS)

Given a complete graph $K_5$

a) draw Two (2) isomorphic graphs of $K_5$.

(2 marks)
b) draw Two (2) subgraphs of $K_5$.

(2 marks)

c) draw Two (2) spanning trees of $K_5$

(2 marks)

d) write the adjacency matrices for $K_5$

(2 marks)
QUESTION 11 (4 MARKS)

a) What is a binary search tree?

(2 marks)

b) Build up the binary tree for the words “She, Sells, Sea, Shells, By, The, Seashore” using the alphabetical order.

(2 marks)

END OF QUESTIONS