UNIVERSITI UTARA MALAYSIA

FINAL EXAMINATION
FIRST SEMESTER SESSION 2008/2009

CODE/COURSE : QQM2053/ PERSAMAAN PEMBEZA
DATE : 15 NOVEMBER 2008 (SATURDAY)
TIME : 2:30 – 4:30 P.M. (2 HOURS)
VENUE : DSB K.T/WD

INSTRUCTIONS:

1. This book script contains TEN (10) questions in TEN(10) printed pages excluding the cover page.
2. Answer ALL the questions in the space provided.

MATRIC NO:______________________________

( in words ) ___________________________
( in numbers ) __________________________

IDENTITY CARD/PASSPORT NO : ___________________________

LECTURER :______________________________

GROUP : ___________________________ TABLE NO.: _____________

DO NOT OPEN THE PAGE UNTIL YOU ARE TOLD TO DO SO

CONFIDENTIAL
QUESTION 1 (4 MARKS)

a) Classify the given differential equations as an ordinary or partial.
   
   i) \( \frac{\partial^2 f}{\partial x^2} + y \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} + \sin y = 0 \)  
   (1 mark)

   ii) \( (\sin z) \frac{dy}{dz} + \frac{\cos y}{z} = 0 \)  
   (1 mark)

b) State the order and name of the dependent and independent variables for
   
   \[ \frac{d^3 p}{dq^3} + \frac{d^2 p}{dq^2} p + 4q^2 = 0. \]  
   (2 marks)

QUESTION 2 (6 MARKS)

Given the first order differential equation \( \frac{dy}{dx} - 3x\sqrt{x^2 + 1} = 0 \)

i) Write the equation in separable form.  
   (2 marks)
ii) Find the general solution.  

QUESTION 3 (10 MARKS)

The function \( y = \ln |xe^x - e^x + C| \) is general solution of \( \frac{dy}{dx} = xe^x - y \) where \( C \) is a constant.

i) Given \( y(1) = 0 \), find \( y(2) \) in four decimal places.  

(3 marks)
ii) Apply Euler’s method to approximate the value of \( y(2) \) with step size \( h = 0.5 \). Give your answer in four decimal places. (6 marks)

iii) Calculate the error at \( x = 2 \). (1 mark)

**QUESTION 4 (12 MARKS)**

Given an exact equation \( \frac{dy}{dx} = \frac{2x - y^3 - pxy^4}{3xy^2 + 20x^2y^3} \).

i) Determine the \( M(x, y) \) and \( N(x, y) \) functions. (2 marks)
Determine the value of $p$ if \( \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \). \hfill (4 \text{ marks})

Find the general solution. \hfill (6 \text{ marks})
QUESTION 5 (6 MARKS)

A chicken roast, initially at 50°F, is placed in a 375°F oven at 5:00 P.M. After 75 minutes it is found that the temperature $T(t)$ of the chicken roast is 125°F. By using Newton’s law of heating,

$$\frac{dT}{dt} = k(375 - T)$$

where $t$ is time. How long does it take to reach 150°F?
QUESTION 6 (15 MARKS)

Given the differential equation and the initial value problem

\[ y'' - 3y' + 2y = 3e^{-x} - 10 \cos 3x; \]
\[ y(0) = 1, \quad y'(0) = 2 \]

Find,

\( i) \) the homogenous solution, \( y_h \). \((4 \text{ marks})\)

\( ii) \) the values of \( C, D \) and \( E \) if \( y_p = Ce^{-x} + D \cos 3x + E \sin 3x \). \((7 \text{ marks})\)
iii) the particular solution, $y(x)$ for the above differential equation.

(4 marks)

QUESTION 7 (4 MARKS)

Based on the above graph,

i) write the unit step function, $f(t)$.

(1 mark)
ii) by using the definition, find the Laplace transforms of \( f(t) \). (3 marks)

**QUESTION 8 (7 MARKS)**

Find the Laplace transforms for each the following:

a) \( f(t) = 3t^2 + t^5 \) (2 marks)

b) \( g(t) = 3e^{2t} \) (2 marks)
c) \[ h(t) = t \sin 6t \]  

(3 marks)

**QUESTION 9 (6 MARKS)**

Find the inverse Laplace transforms for each the following:

a) \[ \frac{12}{s^3} \]  

(2 marks)

b) \[ \frac{1}{4s^2 + 9} \]  

(4 marks)
QUESTION 10 (10 MARKS)

A spring is stretched $1.3 \text{ cm}$ when a $1\text{ kg}$ mass is attached with damping constant $c = 8$. The equation of motion when the spring constant $k = 15$ is given by:

$$y'' + 8y' + 15y = 0$$
$$y(0) = 2, \quad y'(0) = -3.$$  

i) Determine $L\{y'(t)\}$ and $L\{y''(t)\}$.  

ii) Use the Laplace Transform to solve the initial value problem.  

(2 marks) 

(8 marks)