**FINAL EXAMINATION**
**FIRST SEMESTER SESSION 2008/2009**

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<th>CODE/SUBJECT NAME</th>
<th>QQM2043 / KALKULUS II</th>
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<tr>
<td>DATE</td>
<td>17 NOVEMBER 2008 (MONDAY)</td>
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<tr>
<td>TIME</td>
<td>9:00 – 11:00 A.M. (2 HOURS)</td>
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<td>VENUE</td>
<td>DMS</td>
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**INSTRUCTIONS**
1. This book script contains **NINE (9)** questions in **EIGHT (8)** printed pages excluding front page.
2. Answer **ALL** questions in the space provided.

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**MATRIC NO.:**

(in words)

(in numbers)

**IDENTIFICATION NO.:**

**LECTURER:**

**GROUP:**

**TABLE NO.:**

**PLEASE DO NOT OPEN THIS SCRIPT UNTIL YOU ARE TOLD TO DO SO**

**CONFIDENTIAL**
QUESTION 1 (12 MARKS)

Define the following terms:

i) Partial derivative of a function \( f(x, y) \).

(3 marks)

ii) Double integral of \( f(x, y) \) over the region \( R \).

(3 marks)

iii) First order linear differential equations.

(3 marks)

iv) Second order homogeneous differential equation with initial value problem.

(3 marks)
QUESTION 2 (6 MARKS)

Given a function \( f(x, y) = \sqrt{x^2 + y^4} \). Find

i) the domain of \( f(x, y) \). 

\( \) (1 mark)

ii) the range of \( f(x, y) \). 

\( \) (1 mark)

iii) all first partial derivatives 

\( \) (4 marks)

QUESTION 3 (10 MARKS)

Let \( f(x, y) = (x - 2)^2 + (y + 1)^2 + 3 \).

i) Determine and classify the critical points of \( f(x, y) \). 

\( \) (6 marks)
ii) Find the **minimum** and **maximum values** of $f$ in the entire space. Justify your answer.

(2marks)

iii) What are the **minimum** and **maximum values** of $f$ in the interval $0 \leq z \leq 10$?

(2marks)
QUESTION 4 (8 MARKS)

Suppose \( z = \sqrt{xy + y} \), \( x = \cos \theta \) and \( y = \sin \theta \). Evaluate \( \frac{dz}{d\theta} \) when \( \theta = \frac{\pi}{2} \).

QUESTION 5 (5 MARKS)

Calculate the iterated integral \( \int_{2}^{4} \int_{0}^{3} (3 + 2xy) \, dy \, dx \).
QUESTION 6 (6 MARKS)

Given the following integral \( \int_0^1 \int_0^{\sqrt[3]{x}} f(x, y) \, dy \, dx \).

i) Determine the region that was referred to in the above problem. (3 marks)

ii) Express the double integral for the problem by reversing the order of integration. (3 marks)

QUESTION 7 (13 MARKS)

a. Show that \( \frac{dy}{dx} = 1 - x + y - xy \) is separable. Then, solve the equation. (6 marks)
b. Solve \( (2xy^3 + 3y - 3x^2) \, dx + (3x^2 y^2 + 3x) \, dy = 0 \).  

(7 marks)

QUESTION 8 (10 MARKS)

Given the non-homogeneous differential equation \( y'' + y' - 2y = x + \sin x \).

i) Find the homogeneous solution \( y_h \).  

(2 marks)
ii) Using the method of undetermined coefficient, find a suitable trial solution \( y_p \).

(7 marks)

iii) What is the general solution?

(1 mark)
QUESTION 9 (10 MARKS)

Solve $y'' - 6y' + 25y = 0$, $y(0) = -3$ and $y'(0) = -1$. 

END OF QUESTIONS