UNIVERSITI UTARA MALAYSIA

FINAL EXAMINATION
FIRST SEMESTER SESSION 2008/2009

CODE/SUBJECT NAME : QQM2033 / KALKULUS LANJUTAN
DATE : 17 NOVEMBER 2008 (MONDAY)
TIME : 2:30 P.M. – 4:30 P.M. (2 HOURS)
VENUE : BK3(FWB)

INSTRUCTIONS
1. This book script contains SEVEN (7) questions in EIGHT (8) printed pages excluding front page.
2. Answer ALL questions in the space provided.

Q1 Q2 Q3 Q4 Q5
Q6 Q7

TOTAL

MATRIC NO. : __________________________ (in words ) __________________________ (in numbers )

IDENTIFICATION NO.: __________________________

LECTURER: __________________________

GROUP : __________ TABLE NO. : __________

PLEASE DO NOT OPEN THIS SCRIPT UNTIL YOU ARE TOLD TO DO SO

CONFIDENTIAL
QUESTION 1 (10 MARKS)

Let \( \sum_{i=1}^{\infty} \frac{1}{i(i+1)} \) be an infinite series.

i) Find the first, second, and third partial sum of the series. (3 marks)

ii) Prove that the \( n \)th partial sum of the series above is \( S_n = \frac{n}{n+1} \). (5 marks)

iii) Show that \( \sum_{i=1}^{\infty} \frac{1}{i(i+1)} = 1 \). (2 marks)
QUESTION 2 (15 MARKS)

a) Sketch the graphs of polar equations \( r = 3 + 2\cos \theta \) and \( r = 2 \).

(10 marks)
b) Find the area inside $r = 3 + 2 \cos \theta$ and outside $r = 2$. (5 marks)

QUESTION 3 (14 MARKS)

a) Estimate the volume of the solid that lies above the square $R = [0, 2] \times [0, 2]$ and below the elliptic paraboloid $z = 16 - x^2 - 2y^2$ by

i) dividing $R$ into four equal squares and choose the sample point to be the upper right corner of each square $R_{ij}$. (4 marks)
ii) dividing $R$ into four equal squares and choose the sample point to be the midpoint of each square $R_{ij}$.  

(4 marks)

b) Find the exact volume of the solid that lies above the square $R = [0, 2] \times [0, 2]$ and below the elliptic paraboloid $z = 16 - x^2 - 2y^2$.  

(4 marks)

c) From the above results, should you use the upper right corner or midpoint of each square $R_{ij}$ to approximate the volume? Why?  

(2 marks)
QUESTION 4 (12 MARKS)

Determine all relative extrema and saddle points of $f(x, y) = 4xy - x^4 - y^4$. 
QUESTION 5 (7 MARKS)

At what rate is the volume of a box changing if its length is 10 ft and increasing at 3 ft/s, its width is 8 ft and increasing at 2 ft/s, and its height is 4 ft and increasing at 1 ft/s?
QUESTION 6 (10 MARKS)

Evaluate:

a) \( \int_{D} x^2 y^2 \, dA, \quad D = \{(x, y) \, | \, 0 \leq x \leq 2, -x \leq y \leq x \}. \) (4 marks)

b) \( \int_{x}^{1} \int_{y}^{1} \sin(y^2) \, dy \, dx. \) (6 marks)
QUESTION 7 (12 MARKS)

Find the volume of the solid bounded by the four planes $x = 0, y = 0, z = 0$ and $2x + 2y + 2z = 2$ using:

i) Double integrals. (6 marks)

ii) Triple integrals. (6 marks)

- END OF QUESTIONS -