CONFIDENTIAL  QQM2023

UNIVERSITI UTARA MALAYSIA

FINAL EXAMINATION
FIRST SEMESTER SESSION 2008/2009

CODE/SUBJECT NAME : QQM2023 / ALGEBRA LINEAR
DATE : 22nd NOVEMBER 2008 (SATURDAY)
TIME : 2.30 – 4.30 p.m. (2 HOURS)
VENUE : DMS

INSTRUCTIONS:
1. This book script contains TEN (10) questions in NINE (9) printed pages excluding the cover page.
2. Answer ALL questions in the space provided.

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PLEASE DO NOT OPEN THIS SCRIPT UNTIL YOU ARE TOLD TO DO SO

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QUESTION 1 (6 MARKS)

State whether each of the following statements is true or false.

a) Every vector space that is generated by a finite set has a basis.

(..........................)  (2 marks)

b) The length of a vector is equal to the square root of the inner product of the vector with itself.

(..........................)  (2 marks)

c) Let \( L : \mathbb{R}^n \rightarrow \mathbb{R}^n \) be a linear transformation. If \( L(x) = L(x') \), then \( x \) and \( x' \) must be equal.

(..........................)  (2 marks)

QUESTION 2 (11 MARKS)

Let \( \mathbf{u} = (2, 0, -1) \) and \( \mathbf{v} = (3, -4, 1) \).

(i) Find \( 3\mathbf{u} + \mathbf{v} \).  

(2 marks)

(ii) Determine whether \( \mathbf{u} \) and \( \mathbf{v} \) are orthogonal or not. Justify your answer.  

(2 marks)

(iii) Determine the angle between \( \mathbf{u} \) and \( \mathbf{v} \).  

(4 marks)
(iv) Find $u \times v$.  

(3 marks)

**QUESTION 3 (5 MARKS)**

Given $P(3, -1)$ and $Q(0, 3)$.

(i) Find the length of $\overrightarrow{PQ}$  

(3 marks)

(ii) Find a unit vector in the direction of $\overrightarrow{QP}$.  

(2 marks)
QUESTION 4 (12 MARKS)

Given \( S = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} \) and \( v = \begin{bmatrix} 1 \\ 3 \\ -1 \\ 1 \end{bmatrix} \).

(i) Does \( S \) span \( v \)? Why or why not. (10 marks)

(ii) Does \( S \) span \( \mathbb{R}^4 \)? Explain. (2 marks)
QUESTION 5 (5 MARKS)
Determine whether the set of all triangular matrices $\begin{bmatrix} u_{11} & 0 \\ u_{21} & u_{22} \end{bmatrix}$ is a subspace of $M_{22}$.

QUESTION 6 (12 MARKS)
Given a matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$,

(i) determine a basis for row space of $A$.  

(4 marks)
(ii) determine a basis for the column space of $A$.  

(1 mark)

(iii) determine a basis for the nullspace of $A$.  

(5 marks)

(iv) find the rank and nullity of $A$.  

(2 marks)
QUESTION 7 (7 MARKS)

If \( x_1 = \begin{bmatrix} 2 \\ 6 \\ -18 \end{bmatrix}, \ x_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \text{ and } x_3 = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}, \)

(i) show that \( S = \{x_1, x_2, x_3\} \) is an orthogonal basis in \( R^3 \). 

(3 marks)

(ii) determine whether \( S \) is an orthonormal basis or not. Justify your answer. 

(2 marks)

(iii) determine whether \( x_1, x_2 \) and \( x_3 \) are linearly independent or not. Explain. 

(2 marks)
QUESTION 8 (10 MARKS)

Find a matrix $A_{2 \times 2}$ that has eigenvalues $\lambda_1 = 1, \lambda_2 = 2$ and corresponding eigenvectors $x_1 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}, x_2 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ respectively.
QUESTION 9 (4 MARKS)

If $T : V \rightarrow R$ where $T(v_1) = 2$ and $T(v_2) = -3$, find $T(3v_1 + 2v_2)$. Assume that $T$ is a linear transformation.

QUESTION 10 (8 MARKS)

Given $T : R^3 \rightarrow R^4$ where $T(x, y, z) = (x - y + 2z, x + y - z, 2x + z, 2y - 3z)$.

(i) Find $T(4, 1, -3)$.  

(2 marks)
(ii) Find \( \ker(T) \)  

(6 marks)