UNIVERSITI UTARA MALAYSIA

FINAL EXAMINATION
FIRST SEMESTER 2008/2009 SESSION

CODE/COURSE : QQM 1063 / DISCRETE MATHEMATICS
DATE : 10 NOVEMBER 2008 (MONDAY)
TIME : 2.30 P.M. – 5.00 P.M. (2 HOURS 30 MINUTES)
VENUE : DKG 2 / 1

INSTRUCTIONS:

1. This book script contains FIFTEEN (15) questions in TWELVE (12) printed pages excluding the cover page.

2. Answer ALL the questions in the space provided.

MATRIC NO.: __________________ (in words) __________________ (in figures)

IDENTITY CARD NO.: __________________

LECTURER:

GROUP: □ TABLE NO.: [Blank]

DO NOT OPEN THE PAGE UNTIL YOU ARE TOLD TO DO SO

CONFIDENTIAL
QUESTION 1 (5 MARKS)

Circle the best answer.

(a) If \( p \) is true, \( q \) is false and \( r \) is false, then the statement \( ((p \rightarrow (q \lor r)) \land p) \rightarrow r \) is

(A) true  
(B) false

(b) A graph which is not connected must have at least one vertex with degree zero.

(A) true  
(B) false

(c) A tree with \( n \) vertices has \( (n+1) \) edges.

(A) true  
(B) false

(d) A vertex of a tree is called a ________ if it has no children.

(A) isolated  
(B) pendant  
(C) leaf

(e) A finite-state machine with no output is called a ________.

(A) finite-state machine  
(B) finite-state automata

QUESTION 2 (6 MARKS)

Write the negation of each of the following propositions.

(a) \( \forall x \in \mathbb{R}, \ x > 3 \rightarrow x^2 > 9 \)  
[3 marks]
(b) There exists a triangle with the property that the sum of angles is greater than $180^\circ$.

[3 marks]

**QUESTION 3 (7 MARKS)**

Use valid argument to deduce the conclusion from the premises

a) $\neg p \rightarrow r \land \neg s$

b) $t \rightarrow s$

c) $u \rightarrow \neg p$

d) $\neg w$

e) $u \lor w$

\[ \therefore \neg t \]
QUESTION 4 (8 MARKS)

Show that \((p \land q) \lor (p \land \neg q) \lor (\neg p \land \neg q) \equiv p \lor \neg q\) using logical equivalences and/or Boolean identities.

QUESTION 5 (10 MARKS)

Given the Boolean function, \(F(x, y, z) = \overline{xy} + \overline{xz}\).

(a) Draw a logic gate diagram. [5 marks]
QUESTION 6 (6 MARKS)

Suppose G is a graph with vertices $a, b, c, d, e, f$ with adjacency matrix

$$
\begin{bmatrix}
0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 & 1 & 0
\end{bmatrix}
$$

(where alphabetical order is used to determine the rows and columns of the adjacency matrix). Find

(i) the number of vertices in G. [1 mark]

(ii) the number of edges in G. [1 mark]
(iii) the degree of each vertex. [2 marks]

(iv) the number of loops. [1 mark]

(v) the distance between vertex \( a \) and vertex \( e \). [1 mark]

**QUESTION 7 (5 MARKS)**

(a) Draw a graph with four vertices and in which the degree of each vertex is three. What is a special name for this graph. [3 marks]

(c) Draw two subgraphs from question (b). [2 marks]
QUESTION 8 (3 MARKS)

(a) Determine whether $K_3$ is bipartite? Justify your answer. [3 marks]

QUESTION 9 (3 MARKS)

Are these two graphs isomorphic? Justify your answer. [3 marks]
QUESTION 10 (8 MARKS)

(a) State the definition of a tree. [2 marks]

(b) Determine which of the graphs shown below are trees and give reason for the graphs that are not trees. [6 marks]

- G
- H
- J
- K
QUESTION 11 (6 MARKS)

(a) Draw a binary tree to represent \((xy)^2 + \frac{y}{(x + 5)}\).  

[4 marks]

(b) What is the height of this binary tree? Why?  

[2 marks]

QUESTION 12 (8 MARKS)

For the weighted graph shown below, use the Prim’s Algorithm starting with vertex E.
(a) Determine and draw a minimum spanning tree. [6 marks]

(b) Find the total weight. [2 marks]

QUESTION 13 (5 MARKS)

Let $G$ be the phrase-structure grammar with vocabulary $V = \{S, A, B, x, y\}$, the terminal set $T = \{x, y\}$, a start symbol $S$ and the production sets

$P = \{S \to ABx, S \to Bx, A \to xB, AB \to y, B \to xy\}$

(a) Find a derivation of $xyxyyx$. [2 marks]
QUESTION 14 (12 MARKS)

Given is a phrase-structure grammar that generates all multiplications of the form \( a \times b \), where \( a \) is a signed integer and \( b \) is a positive integer. The start sentence is multiplication.

\[
S \rightarrow \text{sign} \text{ integer}
\]

\[
S \rightarrow \text{sign} \text{ integer} \quad \ast \quad \text{positive integer}
\]

\[
\text{sign} \rightarrow +
\]

\[
\text{sign} \rightarrow -
\]

\[
\text{integer} \rightarrow \text{digit}
\]

\[
\text{integer} \rightarrow \text{integer} \quad \text{digit}
\]

\[
\text{digit} \rightarrow i, \ i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 0
\]

\[
\text{positive integer} \rightarrow \text{integer} \quad \text{nonzero digit}
\]

\[
\text{positive integer} \rightarrow \text{integer} \quad \text{nonzero digit} \quad \text{integer}
\]

\[
\text{positive integer} \rightarrow \text{nonzero digit}
\]

\[
\text{nonzero digit} \rightarrow i, \ i = 1, 2, 3, 4, 5, 6, 7, 8, 9
\]
(a) What is the Backus-Naur form (BNF) for this grammar? [6 marks]

(b) Construct a derivation tree for $+318*57$ in this grammar. [6 marks]
QUESTION 15 (8 MARKS)

For the state table below,

<table>
<thead>
<tr>
<th>State</th>
<th>( f )</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a )</td>
<td>( b )</td>
</tr>
<tr>
<td>( s_0 )</td>
<td>( s_0 )</td>
<td>( s_4 )</td>
</tr>
<tr>
<td>( s_1 )</td>
<td>( s_0 )</td>
<td>( s_3 )</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>( s_0 )</td>
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<td>( s_3 )</td>
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<td>( s_4 )</td>
</tr>
<tr>
<td>( s_4 )</td>
<td>( s_1 )</td>
<td>( s_0 )</td>
</tr>
</tbody>
</table>

(a) construct the finite-state automaton. [5 marks]

(b) determine the output where the input string are:

(i) \( ababa \)

(ii) \( baabba \) [2 marks]

(c) determine which input string in question (b) is an accepting state. [1 mark]

END OF QUESTIONS