STQM2103 CONFIDENTIAL

UUM
Universiti Utara Malaysia

FINAL EXAM
SECOND SEMESTER SESSION 2011/2012

COURSE CODE / NAME : STQM2103/DISCRETE STRUCTURE
DATE : 23 JUNE 2012 (SATURDAY)
TIME : 9.00 – 11.30 A.M. (2 ½ HOURS)
VENUE : TE / KTB

INSTRUCTION :

1. This exam paper contains SIXTEEN (16) questions in THIRTEEN (13) printed pages, excluding the cover page.
2. Answer ALL questions in the spaces provided.
3. You are NOT ALLOWED to remove the exam paper from the examination hall.

MATRIC NO : ________________________________ ( with word )

IDENTIFICATION CARD NO. :

LECTURER : ________________________________

GROUP : [ ] TABLE NO. : [ ]

DO NOT OPEN THIS EXAMINATION PAPER UNTIL INSTRUCTED

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STQM2103 DISCRETE STRUCTURE

ANSWER ALL QUESTIONS

1. Let \( A = \{a, b, c, d, e\} \), \( B = \{1, a, b, 2, c\} \) and \( C = \{3, c, a, 1, f\} \).

   a. Find \( A - (B \cap C) \).

   (2 marks)

   b. Draw the Venn diagram to show set \( A, B \) and \( C \) and shade the region of \( A - (B \cap C) \).

   (3 marks)

2. Let \( A = \begin{bmatrix} 3 & -2 & 5 \\ 4 & 1 & 2 \end{bmatrix} \), \( B = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} \), and \( C = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & -1 \\ 2 & 0 & 8 \end{bmatrix} \).

   a. Find \( AB \)

   (2 marks)
b. Find $C^T$  

(3 marks)

3. **Prove the following equation with mathematical induction:**

\[ 8 + 16 + 24 + \ldots + 8n = 4n(n+1) \]  

(7 marks)
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4. Find the number of distinguishable permutations of the letters in:
   a. MEGAMIND

   b. ANGRYBIRDY

   c. SAMSUNGALAXY

   (2 marks)

5. HARRY PORTER would like to perform committees in order to manage his Aerospace company called TINTIN Corp. How many different members of 5 TINTIN committees can be formed, each containing 3 girls from an available set of 10 girls and 2 boys from an available set of 20 boys?

   (3 marks)
6. Let the set $B = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Consider the following subsets of $B$:

$B_1 = \{0, 1, 2, 3\}$, $B_2 = \{0, 2, 4, 5, 6, 7\}$, $B_3 = \{0, 2, 4, 6\}$, $B_4 = \{1, 3\}$, $B_5 = \{5, 7\}$

Determine whether the following sets are partitions of set $B$ and justify your answers:

a. $\{B_3, B_5\}$  
(2 marks)

b. $\{B_3, B_4, B_5\}$  
(2 marks)

7. Given a set of integers $A = \{1, 2, 3\}$, and $R$ is a relation on the set $A$, where $(a, b) \in R$ if and only if $|a - b| \leq 1$.

a. List the ordered pairs in the above relation.  
(2 marks)

b. Draw the digraph of the relation.  
(2 marks)
c. Determine whether the relation is an equivalence relation. (4 marks)

8. Given a set \{1,2,3,4\}, let \( R \) be a relation \{((1,2),(1,3),(2,3),(2,4), (3,1)) \} and let \( S \) be the relation \{((2,1), (3,1), (3,2), (4,2))\).

   a. Find \( R \cup S^{-1} \) (2 marks)

   b. Find \( S \cap \overline{R} \) (2 marks)
9. Based on Pigeonhole Principle, how many students are required in a class to ensure that at least 5 students get the same marks for an exam if the marking scale is 0 to 100? (4 marks)

10. Given the functions \( h(x) = 2x^2 + 2x - 1 \) and \( k(x) = \frac{x+3}{2} \) for \( x = 1, 2, 3, 4, 5 \)
   
   a. Determine the domain and range for \( h(x) \) (3 marks)
b. Find \((k \circ h)(x)\)

(3 marks)

c. Find \((h \circ k)(3)\)

(3 marks)

11. Let \(C = \{1, 4, 5, 8, 9\}\). \(P_1 = \begin{bmatrix} 1 & 4 & 5 & 8 & 9 \\ 5 & 8 & 1 & 9 & 4 \end{bmatrix}\), \(P_2 = \begin{bmatrix} 1 & 4 & 5 & 8 & 9 \\ 4 & 1 & 8 & 9 & 5 \end{bmatrix}\).

Compute:

a. \(P_2^{-1}\)

(3 marks)
b. \[ P_2 \circ P_1 \]  

(3 marks)

c. \[ P_1 \circ P_2^{-1} \]  

(3 marks)

12. Let \( A = \{a, b, c, d, e, f, g, h\} \). The relation \( R = \{(a,b), (a,c), (a,d), (c,h), (d,e), (e,f), (f,g)\} \) is defined on the set \( A \).

a. Draw the tree for the relation \( R \).  

(4 marks)
b. Find the root of the tree for the relation R.  

(2 marks)

c. List all level-2 vertices of the rooted tree for R.  

(2 marks)

d. List all the leaves of the rooted tree for R.  

(2 marks)

13. Determine the order in which the vertices of the ordered rooted tree below are visited using:

![Tree Diagram]

a. An inorder search  

(3 marks)

b. A postorder search  

(3 marks)
c. A preorder search

14. Construct a binary tree for the following algebraic expression:

\[(c - 4) + ( (a + 2) \div 3) \times b)\]
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15. Based on the finite-state machine, please:

   a. Construct the state transition table of the digraph shown.  
(4 marks)
b. Draw the diagraph for the machine with the following state

(3 marks)

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₀</td>
<td>S₁</td>
<td>S₂</td>
<td>S₀</td>
</tr>
<tr>
<td>S₁</td>
<td>S₂</td>
<td>S₀</td>
<td>S₁</td>
</tr>
<tr>
<td>S₂</td>
<td>S₂</td>
<td>S₀</td>
<td>S₁</td>
</tr>
</tbody>
</table>
16. Consider the machine whose state transition table is

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

a. Show that $R= \{(1,1),(1,4),(4,1),(4,4),(2,2),(2,3),(3,2),(3,3)\}$ is a machine congruence (3 marks)

b. Construct the state transition table for the corresponding quotient table (3 marks)